Circular Motion

Rotation Angle

- When an object rotates, all the points along the radius move through the same angle in the same amount of time.
- Therefore, it is convenient to measure position, velocity, and acceleration in terms of angle.

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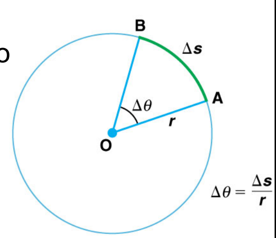
Angular Rotation

- We define the rotation angle $\Delta\theta$ to be the ratio of the arc length to the radius of curvature.

$$\Delta\theta = \frac{\Delta s}{r}$$



The arc length Δs is the distance traveled along a circular path and r is the radius of curvature of the circular path.

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- For one complete revolution, the arc length is the circumference of a circle of radius r.
- The circumference of a circle is $2\pi r$.
- Therefore, for one complete revolution

$$\Delta\theta = \frac{\Delta s}{r} = \frac{2\pi r}{r} = 2\pi$$

- This defines the units we use to measure angular rotation, **radians (rad)**.

$$2\pi \text{ rad} = 1 \text{ revolution} = 360°$$

Angular Velocity

- Rate of change of an angle.

$$\omega = \frac{\Delta\theta}{\Delta t}$$

Units: rad s^{-1}

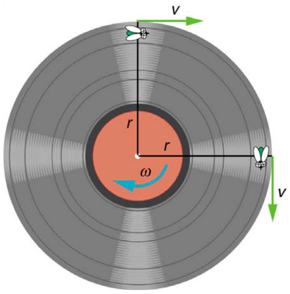
- Angular velocity ω is analogous to linear velocity v.

- A particle moves an arc length of Δs in time Δt.
- The velocity of the object is $v = \frac{\Delta s}{\Delta t}$
- From the definition of angular rotation $\Delta s = r\Delta\theta$
- Substituting gives $v = \frac{r\Delta\theta}{\Delta t} = r\omega$

$$v = \omega r \quad \text{or} \quad \omega = \frac{v}{r}$$

- This relationship tells us two things…

- An object moving with an angular velocity ω has a tangential (linear) velocity at any point is equal to ωr.



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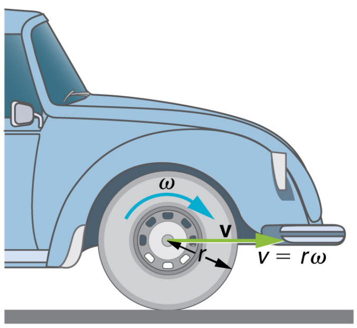
- A rolling object with a linear velocity of v is rotating with an angular velocity of ω.



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Period and Frequency

- The concepts of period and frequency are often used with circular motion.
- The period, T, is the time required for one rotation.
- The frequency, f, is the number of rotations per second.

$$T = \frac{1}{f}$$

- Angular velocity can be expressed in terms of period and frequency.

$$\omega = \frac{v}{r} \qquad v = \frac{x}{t} = \frac{2\pi r}{T}$$

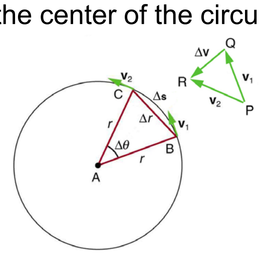
$$\omega = \frac{2\pi}{T} = 2\pi f$$

Due to this relationship, angular velocity is also referred to as angular frequency.

Uniform Circular Motion

- An object that moves in a circle at constant tangential (linear) speed v, is said to experience **uniform circular motion**.
- The speed may be constant but the direction is changing.
- This means that the object is accelerating.
- The acceleration is in the direction of the change in velocity.

- The direction of the acceleration is towards the center of the circular path.



- This acceleration is called **centripetal acceleration** (towards the center or center seeking).

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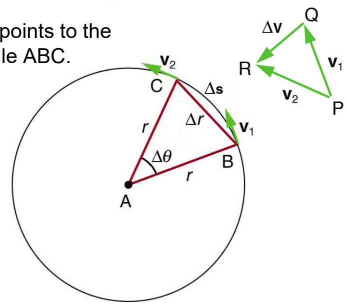
Deriving an Equation for Centripetal Acceleration

- During Δt the object moves from B to C
  - Connecting these points to the center gives triangle ABC.



$$a = \frac{\Delta v}{\Delta t}$$

$$\Delta v = v_2 - v_1$$

- The vector subtraction gives triangle PQR.
  - $v_1 = v_2 = v$

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- The two triangles are similar triangles, therefore

$$\frac{\Delta v}{v} = \frac{\Delta s}{r}$$

- Solving for Δv

$$\Delta v = \frac{v \Delta s}{r}$$

- Divide both sides by Δt

$$\frac{\Delta v}{\Delta t} = \frac{v \Delta s}{r \Delta t}$$

$$\frac{\Delta v}{\Delta t} = a \qquad \frac{\Delta s}{\Delta t} = v$$

$$\boxed{a_c = \frac{v^2}{r}}$$

- It is also useful to express centripetal acceleration in terms of angular velocity, period and frequency.

Substituting $v = r\omega$ into the previous expression $\quad a_c = r\omega^2$

Substituting $\omega = \frac{2\pi}{T}$ $\quad a_c = \frac{4\pi^2 r}{T^2}$

Substituting $f = \frac{1}{T}$ $\quad a_c = 4\pi^2 r f^2$

Example 1

- A car drives around a curve of radius 500.0 m at a speed of 25 m/s. Calculate the centripetal acceleration.

$$a_c = \frac{v^2}{r}$$

$$a_c = \frac{(25)^2}{500}$$

$$a_c = 1.25\ \mathrm{m/s}^2$$

Example 2

- A 150 g ball at the end of a string is revolving in a horizontal circle of radius 0.60 m. The ball makes 2 revolutions in one second. Calculate the centripetal acceleration of the ball.

$$a_c = \frac{v^2}{r} = r\omega^2 = 4\pi^2 r f^2$$

$$a_c = 4\pi^2(0.6)(2)^2$$

$$a_c = 95\ \mathrm{m/s^2}$$

Centripetal Force

- Any force or combination of forces can cause a centripetal or radial acceleration.
- Any net force causing uniform circular motion is called a centripetal force.
- The direction of a centripetal force is toward the center of curvature, the same as the direction of centripetal acceleration.

- According to Newton's second law of motion, $F_{net} = ma$
- For uniform circular motion, the acceleration is the centripetal acceleration $(a = a_c)$.
- Thus, the magnitude of centripetal force is

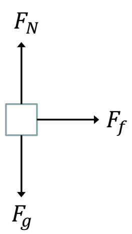
$$F_c = ma_c$$

or

$$F_c = m\frac{v^2}{r} = r\omega^2$$

Example 1

- A 1200 kg car travels around a 500.0 m radius unbanked curve at 25.0 m/s. Calculate the minimum static coefficient of friction between the tires and the road required to keep the car from slipping.



$$F_{net} = ma$$

Since the car is moving in a circle

$$F_{net} = F_c = ma_c$$

$$F_f = ma_c$$

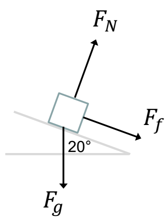
$$\mu F_N = \mu F_g = \mu mg = \frac{mv^2}{r}$$

$$\mu = \frac{v^2}{gr}$$

$$\mu = \frac{(25)^2}{(9.8)(500)} = 0.13$$

Example 2

- A 1200 kg car travels around a 500.0 m radius curve at 25.0 m/s. The curve is banked 20°. Calculate the coefficient of friction between the tires and the road required to keep the car from slipping.



$$F_{net} = ma$$

Since the car is moving in a circle

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